July / 2004
Chad Gillis

A Look at the Effects of Polarizer Cell Curvature on Polarization Measurements Using the Beam Monitors for the NPDGamma Experiment

What this writeup is about

This writeup addresses the effect of polarizer curvature on beam monitor polarimetry for the NPDGamma experiment. We have been performing calculations of neutron and ^{3}He polarization using beam monitor ratios of measurements over the full width of the neutron beam. Such a method can only be exact if the cell is of uniform thickness over the width of the beam. The polarizer cell used during the NPDGamma commissioning run was of nonuniform thickness. It is therefore important to consider the effect of the curvature of the cell on our polarization calculations.

The problem is addressed by deriving mathematical expressions of two different quantities: 1) The actual polarization of the neutron beam after leaving a cell of nonuniform thickness; and 2) the result of the calculation which is done using transmission measurements over the full width of the beam.

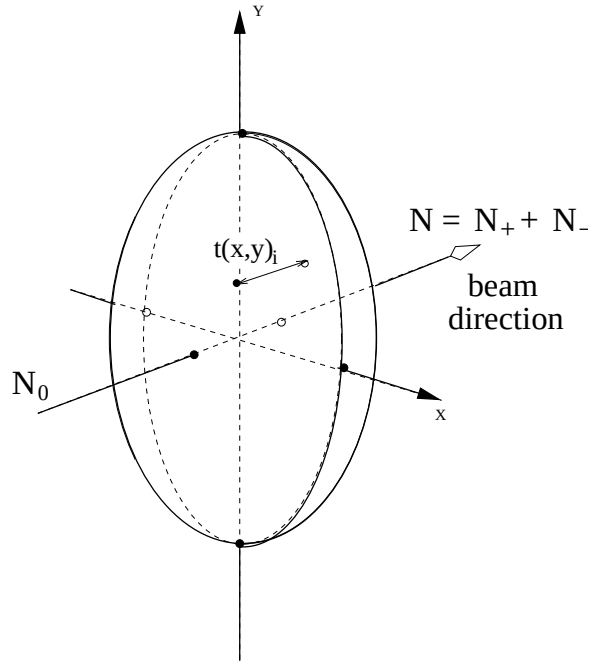
Using reasonable values for the important parameters, a numerical calculation is then done for cases 1) and 2) in order to determine to what extent the results of the two cases differ. It's found that the discrepancy is a factor of a couple tenths of a percent.

The correct expression for neutron polarization

Here we derive the expression for the polarization of the full neutron beam after leaving a polarizer cell of general shape. The beam is assumed to be unpolarized and of uniform neutron density before entering the polarizer.

Consider a polarizer cell which has a nonuniform thickness so that the areal density t of the ^{3}He inside the cell can be expressed as a function of the position of the neutron, as shown in the diagram. The total number of neutrons incident is given by N_0 and the total number of neutrons leaving the cell is given by N.

In order to determine the polarization of the full neutron beam after leaving the cell, first consider some region of the beam which is small enough so that the neutron transmission



can be assumed to be uniform over that region. Allow there to be many such regions (n in total), each subscripted by the index i, which together cover the whole width of the beam. The ith region is centered at the point $(x, y)_i$. The area occupied by the whole beam is given by A while the area occupied by the ith region is given by δA_i.

On leaving the polarizer cell, the number of neutrons pointing in the up direction at point $(x, y)_i$ is then given by

$$\delta N_+^i = \frac{\delta A_i}{A}\ N_0\ exp(-t(x,y)_i\ [1 - P_3]\ \sigma) \tag{1}$$

and the number of neutrons pointing in the down direction is given by

$$\delta N_-^i = \frac{\delta A_i}{A}\ N_0\ exp(-t(x,y)_i\ [1 + P_3]\ \sigma). \tag{2}$$

If $N_+ = \sum \delta N_+^i$ is the total number of neutrons pointing in the up direction and $N_- = \sum \delta N_-^i$ is the total number of neutrons pointing in the down direction over the whole beam downstream of the polarizer cell, then the polarization of the beam as a whole is given by

$$\begin{aligned} P = \frac{N_+ - N_-}{N_+ + N_-} &= \frac{\sum[\frac{\delta A_i}{A} N_0 exp(-t(x,y)_i[1 - P_3]\sigma) - \frac{\delta A_i}{A} N_0 exp(-t(x,y)_i[1 + P_3]\sigma)]}{\sum[\frac{\delta A_i}{A} N_0 exp(-t(x,y)_i[1 - P_3]\sigma) + \frac{\delta A_i}{A} N_0 exp(-t(x,y)_i[1 + P_3]\sigma)]} \\ &= \frac{\sum \delta A_i exp(-t(x,y)_i\sigma)[exp(t(x,y)_i P_3\sigma) - exp(-t(x,y)_i P_3\sigma)]}{\sum \delta A_i exp(-t(x,y)_i\sigma)[exp(t(x,y)_i P_3\sigma) + exp(-t(x,y)_i P_3\sigma)]} \end{aligned}$$

$$P = \frac{\sum \delta A_i exp(-\sigma t(x,y)_i) sinh(\sigma t(x,y)_i P_3)}{\sum \delta A_i exp(-\sigma t(x,y)_i) cosh(\sigma t(x,y)_i P_3)} \tag{3}$$

Letting $n \to \infty$:

$$P = \frac{\int exp(-\sigma t(x,y))sinh(\sigma t(x,y)P_3)dA}{\int exp(-\sigma t(x,y))cosh(\sigma t(x,y)P_3)dA} \tag{4}$$

An expression for the neutron polarization as calculated from the beam monitor ratios

We will now derive an expression for the neutron and ^{3}He polarizations that we're calculating using the beam monitor ratios. These quantities will be referred to as P' and P_3' respectively in order to distinguish them from the true polarizations P and P_3.

Using beam monitor data, the following expression has been used to determine the neutron and ^{3}He polarizations:

$$P' = tanh(\sigma t' P_3') = \sqrt{1 - \left(\frac{T_0}{T_P}\right)^2}. \tag{5}$$

Here T_0 is the transmission of the full beam through the ^{3}He when it's unpolarized and T_P is the transmission of the full beam through the ^{3}He at the time that the polarization measurement is being made. t' is the areal density of ^{3}He as determined by a full beam transmission measurement according to the expression:

$$T_0 = exp(-\sigma t'). \tag{6}$$

The number of neutrons that make it to the other side of the cell for an infinitesimal element covering an area δA_i of the cell is:

$$\delta N_i = \frac{N_0}{A}\delta A_i \; exp(-\sigma t(x,y)_i) \quad \text{(unpolarized)} \tag{7}$$

and

$$\delta N_i = \frac{N_0}{A}\delta A_i \; exp(-\sigma t(x,y)_i)cosh(\sigma t(x,y)_i P_3) \quad \text{(polarized)}. \tag{8}$$

The transmission of the full beam, whether polarized or unpolarized, is given by $T = \frac{N}{N_0}$ where $N = \sum \delta N_i$. This yields:

$$T_0 = \frac{1}{A}\int exp(-\sigma t(x,y))dA \tag{9}$$

and

$$T = \frac{1}{A}\int exp(-\sigma t(x,y))cosh(\sigma t(x,y)P_3)dA. \tag{10}$$

Therefore, according to equation (5):

$$P' = \sqrt{1 - \left(\frac{\int exp(-\sigma t(x,y))dA}{\int exp(-\sigma t(x,y))cosh(\sigma t(x,y)P_3)dA} \right)^2}. \tag{11}$$

P_3' can then also be determined from equation (5).

The Calculations

In order to be able to evaluate the difference between P and P', some assumptions about the properties of the cell need to be made. First a crude model of the cell dimensions will be assumed. This model can be refined later if deemed necessary by making measurements of the shape of the cell.

At NIST, the thickness at the center of the cell was determined to be 5.5 Atm.cm and at LANSCE t' was determined from beam monitor measurements to be 4.85 Atm.cm.

A simple model is to assume that the two walls of the cell are conical in shape. The cell would then have a peak thickness in the center of 5.5 Atm.cm and the slope of the walls would be determined in order to give $t' = 4.85$ Atm.cm. This corresponds to the following expression for t:

$$t = 5.5 \text{ Atm.cm} - (0.172 \text{ Atm})r \tag{12}$$

where r is the perpendicular distance from the cell axis.

The rest of the required assumptions are:

- L = distance from source to M2 = 21.40 m.
  (Measurements of cell transmission are made with M2)
- ^{3}He polarization = 46 %.
- Neutron beam is 9.5 cm x 9.5 cm square and of constant neutron density

With these assumptions in mind, the integrals in equations (4) and (11) were calculated numerically in order to deterimine P and P'. P_3' was then determined according to equation (5) as:

$$P_3' = \frac{arctanh(P')}{\sigma t'}. \tag{13}$$

Each of the calculations mentioned above were done for various times of flight, as shown in the table below.

tof (ms)	t′ (Atm.cm)	P	P'	P'/P	P'_3	P'_3 / P_3
2.0	4.8570	0.0660	0.0659	1.0011	0.4605	1.0010
4.0	4.8562	0.1312	0.1310	1.0011	0.4604	1.0008
6.0	4.8554	0.1952	0.1950	1.0011	0.4603	1.0007
8.0	4.8546	0.2576	0.2573	1.0011	0.4602	1.0005
10.0	4.8538	0.3179	0.3175	1.0011	0.4602	1.0004
12.0	4.8530	0.3756	0.3752	1.0011	0.4601	1.0002
14.0	4.8522	0.4306	0.4301	1.0011	0.4600	1.0001
16.0	4.8514	0.4825	0.4820	1.0010	0.4600	0.9999
18.0	4.8506	0.5312	0.5306	1.0010	0.4599	0.9998
20.0	4.8498	0.5766	0.5760	1.0010	0.4598	0.9997
22.0	4.8490	0.6187	0.6181	1.0010	0.4598	0.9995
24.0	4.8482	0.6574	0.6568	1.0009	0.4597	0.9994
26.0	4.8475	0.6930	0.6924	1.0009	0.4597	0.9993
28.0	4.8467	0.7255	0.7249	1.0009	0.4596	0.9992
30.0	4.8459	0.7550	0.7544	1.0008	0.4596	0.9990
32.0	4.8451	0.7817	0.7811	1.0008	0.4595	0.9989
34.0	4.8443	0.8059	0.8053	1.0007	0.4595	0.9988
36.0	4.8436	0.8276	0.8270	1.0007	0.4594	0.9987
38.0	4.8428	0.8470	0.8465	1.0007	0.4594	0.9986
40.0	4.8420	0.8645	0.8639	1.0006	0.4593	0.9985

Conclusion

According to the model used in this writeup, corrections to neutron polarization due to nonuniformities in the thickness of the polarizer cell are a factor of 0.1 % or less. Similar corrections to the ^{3}He polarization are a factor of 0.2 % or less.